INVESTIGATION OF THE PROCESS OF JET BURNING OF GASEOUS FUEL

A. L. Rutkovskii, E. I. Meshkov, A. M. Davidson, A. T. Zurabov, and M. A. Kovaleva

UDC 669.041:536.2.001.537

The process of burning-out of the gaseous fuel jet in a wide range of variation of the influencing parameters has been investigated. We have obtained a differential equation of one-dimensional jet burning-out and its solution, on the basis of which we have derived equations for calculating the degree of burn-out and the jet length. The influence of the change in the theoretical volume of oxygen has been investigated, which permits justified calculations of the gaseous fuel burn-out in industrial units and control of the process technology. The results obtained have been used to modernize the gas burners of alumina calcination furnaces, which permits substantial savings on fuel.

Keywords: *gaseous fuel, jet length, degree of burn-out, influencing parameters, industrial units.*

Introduction. The design and operating parameters of thermal technological units used in metallurgy are characterized by a wide spread, which impedes comparative analysis of their operation. At the present time, the majority of industrial furnaces use gaseous fuel. However, the questions of its combustion are still not clearly understood, which makes it impossible to analyze the process of fuel burn-out in the jet and, therefore, the laws of the temperature distribution of the gas and the material along the length of the furnaces, as well as the conditions for optimizing the process of fuel burning. As a result, in calculations the jet burn-out characteristics are specified by approximate empirical equations, which lowers the accuracy of calculations and leads finally to thermal energy losses in industrial units. For example, in [1] the following approximate equation was proposed for calculating the fraction of unburnt solid fuel at a certain length of the jet:

$$\Omega = 1 - \alpha \left(1 - \exp \left(- \frac{Px}{\alpha} \right) \right). \tag{1}$$

Equation (1) gives a good approximation only at a small length of the jet (on its initial segments), since with increasing x the value of Ω (at $\alpha > 1$) quickly becomes negative, which contradicts the physics of the process.

The possibility of solving the problem on burn-out of solid and liquid fuels on the basis of the investigation of the mass transfer processes in the elementary volume of a one-dimensional jet at analogous approximations was shown in [2], where equations coinciding in form with the equations given in [1] were obtained. This made it possible to draw conclusion on the possibility of an analogous approach to the solution of the problem on burn-out of gaseous fuel obtained in [3] under the same assumptions and significant restrictions.

The present paper is devoted to the investigation of the process of burning-out of a gaseous fuel jet in a wide range of change in parameters, which will make it possible to justifiably approach the thermal calculations of industrial units. The large ratio of the length of the jet to its cross section permits modeling the jet in a one-dimensional space.

Formulation of the Problem. In the jet burning of solid fuels the fraction of oxygen $C_{O_2}^{in}$ at the start of the jet is the same for all fuels and equal to the fraction of oxygen in the draft [2]

North-Caucasus Mining-Metallurgy Institute (State Technological University), RSO-Alaniya, 44 Nikolaev Str., Vladikavkaz, 362000, Russia; email: Rutkowski@mail.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 82, No. 1, pp. 134–140, January–February, 2009. Original article submitted June 10, 2008.

$$C_{O_2}^{in} = \frac{\alpha L_{O_2}^{t}}{\alpha \frac{L_{O_2}^{t}}{\beta}} = \beta . \tag{2}$$

As opposed to this, in the jet burning of gaseous fuel the volume fraction of oxygen at the start of the jet is

$$C_{O_2}^{in} = \frac{\alpha L_{O_2}^{t}}{1 + \alpha \frac{L_{O_2}^{t}}{\beta}} = \frac{\alpha}{\frac{1}{L_{O_2}^{t}} + \frac{\alpha}{\beta}} . \tag{3}$$

Consequently, for gaseous fuel this quantity is variable, depending on the excess draft coefficient, the fraction of oxygen in the draft and its rate depending on the fuel composition. This is the fundamental difference of the process of jet burning of gaseous fuels from the process of burning solid fuels, which was first noted in [3].

Taking into account the results of [3], let us represent the process of mass transfer in an infinitely small volume located at distance x from the jet source by the following scheme: SW is the flow rate of the gaseous fuel arriving through the cross-section S into the elementary volume; $SW + S\frac{\partial W}{\partial x}dx$ is the flow rate of the gaseous fuel leaving the elementary volume; $SWPC_{O_2}^{cur}dx$ is the flow rate of burned-out gaseous fuel at length dx. Then the condition of burning-out and continuity of the gas flows and the draft in the one-dimensional jet is described by the following equation:

$$SW = SW + S\frac{\partial W}{\partial x}dx + SPWC_{O_2}^{cur}dx . \tag{4}$$

Mutual cancellation of SW, reduction by S and dx, and replacement of the partial derivative by the total one yield the differential equation for the rate of change in the fuel flow density along the length of the one-dimensional jet:

$$\frac{dW}{dx} + PWC_{O_2}^{cur} = 0 . \tag{5}$$

The current value of the volume fraction of oxygen is given by the following formula:

$$C_{O_2}^{cur} = \frac{V_{O_2}^{cur}}{V_{cur}} = \frac{\alpha_{cur} L_{O_2}^{t}}{1 + \alpha \frac{L_{O_2}^{t}}{\beta}} \frac{W}{W_{in}} = \frac{\alpha_{cur}}{\frac{1}{L_{O_2}^{t}} + \frac{\alpha}{\beta}} \frac{W}{W_{in}} . \tag{6}$$

Note that at the start of the jet $\alpha_{cur} = \alpha$. Consequently, $\lim\limits_{W \to W_0} C_{O_2}^{cur}$ coincides with (3). To eliminate the variable quantity α_{cur} from formula (6), let us express its value in terms of α:

$$\alpha_{cur} = \frac{V_{O_2}^{cur}}{WL_{O_2}^{t}} = \frac{V_{O_2}^{in} - V_{O_2}^{exp}}{WL_{O_2}^{t}} = \frac{\alpha L_{O_2}^{t} W_0 - L_{O_2}^{t}(W_0 - W)}{WL_{O_2}^{t}} = (\alpha - 1)\frac{W_{in}}{W} + 1 . \tag{7}$$

Figure 1 shows the dependences of α_{cur} on the value of the ratio W_{in}/W at various values of α. Proceeding from these dependences, it would be expected that the solution of Eq. (5) will have a different character for $\alpha = 1$ and for $\alpha > 1$, since from (7) it follows that at $\alpha > 1$ $\lim\limits_{W \to 0} \alpha_{cur} = \infty$, and at $\alpha = 1$ α_{cur} is a constant equal to one.

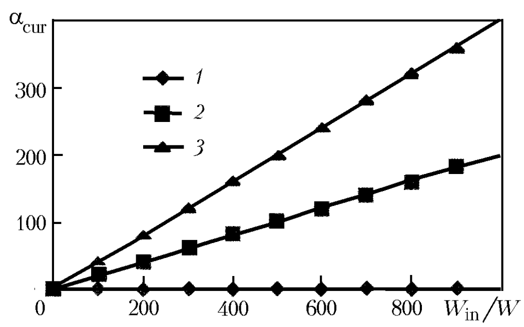


Fig. 1. Dependence of α_{cur} on the ratio W_{in}/W: 1) $\alpha = 1$; 2) 1.2; 3) 1.4.

Upon substitution in (6) of the variable α_{cur} by its dependence on α in accordance with (7) the current concentration of oxygen in the draft will be defined as follows:

$$C_{O_2}^{cur} = \frac{V_{O_2}^{cur}}{V_{cur}} = \frac{(\alpha - 1)\, W_{in} + W}{\left(\frac{1}{L_{O_2}^{t}} + \frac{\alpha}{\beta}\right) W_{in}} \,. \tag{8}$$

From this expression it is seen that at $\alpha > 1$

$$\lim_{W \to 0} C_{O_2}^{cur} = \frac{\alpha - 1}{\frac{1}{L_{O_2}^{cur}} + \frac{\alpha}{\beta}} \,, \tag{9}$$

and at $\alpha = 1$

$$\lim_{W \to 0} C_{O_2}^{cur} = 0 \,.$$

Let us substitute the value of $C_{O_2}^{cur}$ into Eq. (5) and after elementary manipulations we will obtain the differential equation for the fuel burn-out in the one-dimensional jet:

$$\frac{dW}{dx} = -\frac{P\,[(\alpha - 1)\, W_{in} + W]}{\left(\frac{1}{L_{O_2}^{t}} + \frac{\alpha}{\beta}\right) W_{in}}\, W \tag{10}$$

or

$$\frac{dW}{\frac{1}{W_{in}} W^2 + (\alpha - 1)\, W} = -\frac{Pdx}{\frac{1}{L_{O_2}^{t}} + \frac{\alpha}{\beta}} \,. \tag{11}$$

To solve the differential equation (11), we used the program product for mathematical calculations MathCad-11 Pro [4]. With its aid the integral on the left-hand side of (11) was calculated as follows:

$$\int \frac{dW}{\frac{1}{W_{in}} W^2 + (\alpha - 1)\, W} = \frac{1}{\alpha - 1} \ln (W) - \frac{1}{\alpha - 1} \ln \left[\frac{W}{W_{in}} + \alpha - 1\right]. \tag{12}$$

Consequently, the general solution of the differential equation (10) is of the form

$$\frac{1}{\alpha-1}\ln(W)-\frac{1}{\alpha-1}\ln\left[\frac{W}{W_{in}}+(\alpha-1)\right]=-\frac{P}{\dfrac{1}{L^{t}_{O_2}}+\dfrac{\alpha}{\beta}}x+C\,. \tag{13}$$

It can also be obtained analytically. The integration constant C has been found from the initial conditions: at $x = 0$, $W = W_{in}$. Then $C = \frac{1}{\alpha-1}\ln(W_{in})-\frac{1}{\alpha-1}\ln(\alpha)$. Substituting the value of C into (13), upon algebraic manipulations we obtain the equation of gaseous fuel burn-out for the case of $\alpha > 1$:

$$W=W_{in}\frac{\alpha-1}{\alpha\exp\left[\dfrac{P(\alpha-1)}{\dfrac{1}{L^{t}_{O_2}}+\dfrac{\alpha}{\beta}}x\right]-1}\,. \tag{14}$$

For the case where the fuel is burned at $\alpha = 1$, Eq. (11) will have the following form:

$$\frac{dW}{W^2/W_{in}}=-\frac{P}{\dfrac{1}{L^{t}_{O_2}}+\dfrac{\alpha}{\beta}}dx\,, \tag{15}$$

whose solution expresses the dependence of the fuel flow density on the jet length for these conditions:

$$W=W_{in}\frac{1}{\dfrac{Px}{\dfrac{1}{L^{t}_{O_2}}+\dfrac{\alpha}{\beta}}+1}\,. \tag{16}$$

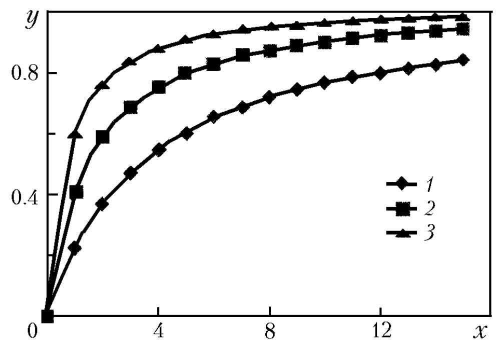
Equation (16) is mainly of theoretical interest, since in practice the fuel is actually burned with $\alpha > 1$.

Results and Discussion. The coefficient P entering into the obtained equations depends on the combustion kinetics and the flow rate of the fuel, as well as on the content in it of combustibles and on its burning conditions, namely: the design of the gas burner and the method of preparing the gas-air mixture for burning. If the gas-air mixture at the outlet from the burner is mixed well, then the P value will be fairly large and the process of combustion will proceed under the most favorable conditions from the point of view of heat absorption in the metallurgical unit — in the kinetic regime [5]. But if the mixture is not mixed well enough, then the combustion process proceeds in the diffusion regime. In so doing, the P value is relatively small and the combustion process is retarded, which leads to an elongation of the jet and a decrease in its maximum temperature.

Using Eq. (14), we have determined the portion of the flow of the fuel y burned-out in the jet to the cross section x from the start of the jet:

$$y=\frac{W_{in}-W}{W_{in}}=1-\frac{\alpha-1}{\alpha\exp\left[\dfrac{Px}{\dfrac{1}{L^{t}_{O_2}}+\dfrac{\alpha}{\beta}}(\alpha-1)\right]-1}\,. \tag{17}$$

If from (17) we express the jet length in terms of y, then we obtain the equation



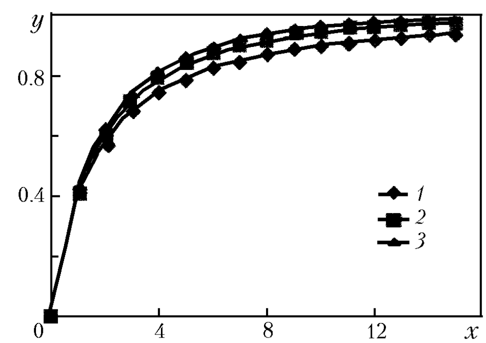


Fig. 2. Influence of the macroconstant P on the degree of burn-out of the jet y: 1) P = 1.5; 2) 3.5; 3) 7.5.

Fig. 3. Influence of the draft excess coefficient α on the degree of burn-out of the jet y: 1) α = 1.1; 2) 1.3; 3) 1.5.

$$x = \frac{\ln\left[\frac{\alpha - y}{\alpha\,(1 - y)}\right]}{\frac{P}{\frac{1}{L^{t}_{O_2}} + \frac{\alpha}{\beta}}\,(\alpha - 1)}\,. \tag{18}$$

From (18) it follows that complete fuel burnup (y = 1) is reached at a jet length $x = \infty$, which does not contradict the theoretical postulates of [5, 6]. Therefore, for practical calculations it is necessary to specify a rather small fraction, e.g., η = 0.05, of incompletely burned fuel and determine the jet length corresponding to this quantity. From (18) we find that under these conditions the total length of the jet is

$$L_{j} = \frac{\ln\left(\frac{\eta + \alpha - 1}{\eta\alpha}\right)}{\frac{P}{\frac{1}{L^{t}_{O_2}} + \frac{\alpha}{\beta}}\,(\alpha - 1)}\,. \tag{19}$$

Relations (14)–(19) represent new results as compared to the analogous equations in [1–3] under the approximate initial assumptions made there, making it impossible to find the exact solution of the problem.

To estimate the numerical values of the parameters of the dependences obtained, we have made experimental studies on calcination furnaces in the alumina production. It has been found that for a certain type of fuel and different designs of burners the values of the coefficient P are in the 1.5–1.7 range. For example, for P = 3.5, α = 1.1, $L^{t}_{O_2}$ = 2.0, β = 0.21, η = 0.05, in accordance with (19) we find L_{j} = 16.5 m, which corresponds to the practical operation of these furnaces [3]. This has made it possible to investigate the influence of the main parameters on the indices of the process of fuel burn-out.

Having given several values of the macroconstant P and holding constant the other parameters, we obtain the relation between the degree of burn-out and the jet length at different values of P (Fig. 2). The shape of the curves in Fig. 2 supports the initial premise that an increase in the parameter P, all other things being equal, leads to a decrease in the jet length, i.e., the rate of combustion increases thereby.

Figure 3 presents the degree of combustion as a function of the jet length at a different initial draft excess coefficient for P = 3.5. The other parameters are held constant. It is seen from Fig. 3 that an increase in the initial draft excess coefficient, all other things being equal, leads to an increase in the rate of jet burn-out. Analogous results

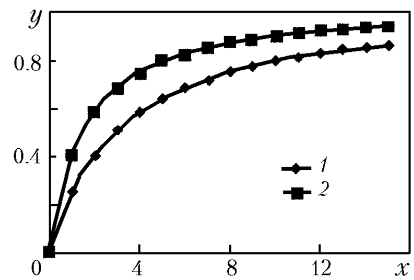


Fig. 4. Influence of the kind of fuel on the process of burn-out of the jet: 1) blast furnace gas ($L^{t}_{O_2}$ = 0.156); 2) natural gas ($L^{t}_{O_2}$ = 1.861).

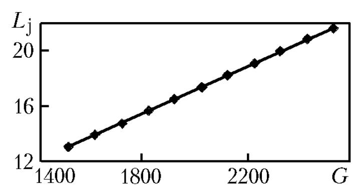


Fig. 5. Jet length L_j a a function of the fuel rate G at the calculated value of the macroconstant of the fuel combustion reaction rate k = 7389 m^2/h under the experimental conditions on the alumina calcination furnace. L_j, m; G, m^3/h.

have been obtained in investigating the influence of an increase in the content of oxygen in the draft, since the influence of these factors is actually identical.

We have also investigated the influence of the change in the flow rate of oxygen ($L^{t}_{O_2}$) on the jet burn-up rate and length. Two kinds of fuel were compared: natural gas, for which $L^{t}_{O_2}$ = 1.861 m^3/m^3, the initial fraction of oxygen, according to (4), $C^{in}_{O_2}$ = 0.19, and the sum of combustible components was 92.78%, and blast-furnace gas having $L^{t}_{O_2}$ = 0.156 m^3/m^3, $C^{in}_{O_2}$ = 0.095, and the sum of combustible components of 30.143%.

The results of the calculations are given in Fig. 4. They show that the low-calorific fuel (blast-furnace gas), all other things being equal, burns out much more slowly than the high-calorific gas (natural gas), which is likely to be due to the different initial concentration of oxygen in the gas-air mixture because of the high content of inert components (CO_2, H_2O) in the blast-furnace gas (see (4)).

With the help of the Kantorovich notion on the constant P for solid fuel [1] this constant for the gaseous fuel can be interpreted as follows:

$$P = \frac{k\,(1 - m)}{G}\,, \qquad (20)$$

where the constant m, instead of the porosity of the solid fuel flow, as is assumed in [1], will imply the fraction of incombustible components in the gaseous fuel. Then, having an experimentally measured length of the jet, at a certain fuel rate G from (19) and (20) one can determine the numerical value of the macroconstant of the reaction rate of fuel combustion k, which in turn makes it possible to find the dependence of the total length of the jet on the fuel rate for the investigated furnace. This dependence for the above experimental conditions and at m = 0.1 has the form shown in Fig. 5. Its application is indispensable for qualitative control of the course of the technological process [7].

The laws obtained in the present paper make it possible to investigate the influence of the basic factors on the process of combustion of a gaseous jet and justifiably approach the calculations of its burn-out in industrial units. However, it should be borne in mind that they have mainly a qualitative character, since for obtaining a precise picture of the process the equations of gaseous fuel burn-out presented in this paper should be solved simultaneously with the equations of heat transfer in a particular unit [6]. The results of the investigations performed have made it possible to justifiably modify the burners on alumina calcination furnaces at the aluminum plant (Krasnoturinsk, Sverdlov region) with the aim of increasing the macroconstant P [8]. This permitted a 6–12% decrease in the fuel rate [9].

Conclusions. We have obtained new results on the investigation of certain basic laws of the process of gaseous fuel jet burn-out in a wide range of variation of influencing parameters. A differential equation of the process of fuel burn-out in a one-dimensional jet has been proposed and solved. On the basis of this solution, equations for calculating the degree of fuel burn-out and the jet length have been derived. The influence of the basic parameters of the jet on the degree of fuel burn-out has been investigated, which makes it possible to justifiably approach the calculations of the gaseous fuel burn-out in industrial units. The equations of gaseous fuel burn-out presented in the paper should be solved simultaneously with the equations of heat transfer in a particular unit.

NOTATION

$C_{O_2}^{in}$, initial value of the volume fraction of oxygen in the jet (at $x = 0$); $C_{O_2}^{cur}$, same, current value (at $x > 0$); G, fuel rate, m^3/h; k, macroconstant of the rate of combustion of fuel, m^2/h; L_j, total length of the jet at which a portion of unburned fuel η remained, m; $L_{O_2}^{t}$, theoretically indispensable volume of oxygen per 1 m^3 of gaseous fuel, m^3/m^3; m, fraction of incombustible components in the fuel; P, coefficient characterizing the linear rate of fuel burn-out (macroconstant), 1/m; S, cross-section of the jet, m^2; $V_{O_2}^{in}$, initial value of the flow density of oxygen in the jet, $m^3/(sec \cdot m^2)$; $V_{O_2}^{exp}$, same, expended; $V_{O_2}^{cur}$, same, current; V_{cur}, current flow density of the jet gases, $m^3/(sec \cdot m^2)$; W_{in}, initial volume flow density of gaseous fuel, $m^3/(sec \cdot m^2)$; W, volume flow density of gaseous fuel, $m^3/(sec \cdot m^2)$; x, current length of the jet, m; y, portion of the flow of the fuel burned-out in the jet to the cross-section x; α, draft excess coefficient; α_{cur}, current value of the draft excess coefficient; β, volume fraction of oxygen in the draft; Ω, portion of unburned solid fuel on length x; η, portion of unburned gaseous fuel at the total length of the jet. Subscripts and superscripts: i, initial; exp, expended; cur, current; t, theoretical; j, jet.

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